REPORT DOCUMENTATION PAGE	AFRL-SR-AR-TR-06-0466

The public reporting burden for this collection of information is estimated to average 1 hour per response, including th gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments re information, including suggestions for reducing the burden, to the Department of Defense, Executive Services and Comn that notwithstanding any other provision of law, no person shall be subject to any penalty for failing to comply with a c control number.

PLEASE DO NOT RETURN YOUR FORM TO THE ABOVE ORGANIZATION.

1. REPORT DATE (DD-MM-YYYY)	2. REPORT TYPE	3. DATES COVERED (From - To)
31 OCT 06	FINAL REPORT	1 MAY 04 TO 31 APR 06

4. TITLE AND SUBTITLE	5a. CONTRACT NUMBER
HUMAN PROBLEM SOLVING - AN EXTENSION OF NEWELL & SIMON'S PARADIGM	
	5b. GRANT NUMBER FA9550-04-1-0297
	5c. PROGRAM ELEMENT NUMBER 61102F
6. AUTHOR(S) PROF ZYGMUNT PIZLO	5d. PROJECT NUMBER 2313
	5e. TASK NUMBER BX
	5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)	8. PERFORMING ORGANIZATION REPORT NUMBER
PURDUE UNIVERSITY DEPARTMENT OF PSYCHOLOGICAL SCIENCES WEST LAFAYETTE, IN 47907-2081	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	**10. SPONSOR/MONITOR'S ACRONYM(S)**
AFOSR/NL 875 NORTH RANDOLPH STREET SUITE 325, ROOM 3112 ARLINGTON, VA 22203-1768 Dr Jerome Busemeyer/NL	11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT

APPROVE FOR PUBLIC RELEASE: DISTRIBUTION UNLIMITED

13. SUPPLEMENTARY NOTES

14. ABSTRACT

Several psychophysical studies on human problem solving were performed. These studies involved the following problems: the Traveling Salesman Problem, the 15-puzzle and variants of this puzzle with different sizes, and finally, the TSP with obstacles. All these problems are difficult combinatorial problems and are considered intractable. However, human subjects were found to produce near-optimal solutions very quickly. For all these problems, a pyramid algorithm was used as a model of the mental representation of the problem, and of the global-to-local process of producing the solution. This approach represents a paradigm shift in the study of human problem solving because the prior research (i) neglected mental representation of problems and (ii) it concentrated on easy problems that are not solved well by human. As a part of this project, a workshop on human problem solving was held at Purdue. After the workshop, a new journal was launched. This is the first journal dedicated to human problem solving.

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT	b. ABSTRACT	c. THIS PAGE			19b. TELEPHONE NUMBER (Include area code)

Final Report, October 31, 2006
AFOSR

PI: Zygmunt Pizlo
Department of Psychological Sciences
Purdue University
West Lafayette, IN 47907-2081

Title: Human Problem Solving – an Extension of Newell & Simon's Paradigm

Agreement Number: FA9550-04-1-0297

Start Date: May 1, 2004.

Abstract.

Several psychophysical studies on human problem solving were performed. These studies involved the following problems: the Traveling Salesman Problem, the 15-puzzle and variants of this puzzle with different sizes, and finally, the TSP with obstacles. All these problems are difficult combinatorial problems and are considered intractable. However, human subjects were found to produce near-optimal solutions very quickly. For all these problems, a pyramid algorithm was used as a model of the mental representation of the problem, and of the global-to-local process of producing the solution. This approach represents a paradigm shift in the study of human problem solving because the prior research (i) neglected mental representation of problems and (ii) it concentrated on easy problems that are not solved well by humans. As a part of this project, a workshop on human problem solving was held at Purdue. After the workshop, a new journal was launched. This is the first journal dedicated to human problem solving.

Table of contents.

Introduction.

Scientific research on human thinking and problem solving started around the time of the Gestalt Revolution. Gestalt psychologists emphasized the role of organizing principles in both perception and thinking. In perception, the organizing principles took the form of a simplicity principle determining figure-ground organization and perceived shape. In problem solving, the organizing principles led to the concept of insight. In both perception and thinking, experimental data came from introspection and verbal protocols. This kind of qualitative information seemed sufficient to demonstrate the operation of the organizing principles. However, it was not sufficient to study the nature of the underlying mental mechanisms. In order to learn what information is used and how it is analyzed, one has to perform parametric studies, in which behavioral response (accuracy and response times) is measured, while the nature of the stimulus is systematically varied. Parametric studies were not news in perception: the concept of threshold had been known for about a century before Gestalt Revolution. However, parametric studies were, and still are news in thinking and problem solving. Problems, unlike physical objects, are not represented by continuous variables. For example, the size of an object can be manipulated with an arbitrary precision and one can produce many objects that are identical except for size. However, physics and math problems are best represented by graphs and one problem cannot be changed to another with arbitrarily small steps. This makes parametric studies difficult. A given subject is usually tested with only one instance of a given physics or math problem. But performance (accuracy and response time) obtained from one instance is insufficient to infer the underlying mental mechanisms. Introspection and verbal protocol seemed the only way to go.

This state of affairs started to change a decade ago, when the interest of cognitive psychologists shifted to optimization problems, such as the Traveling Salesman Problem (TSP). TSP is defined as follows: given a set of points (called cities), find a tour of the points with the *shortest* length. A tour is a path which passes through each point once, and returns to the starting point. The number of tours in a problem with N points is (N-1)!/2. Clearly, finding a shortest tour is an optimization problem. This problem is difficult because the number of tours is large, even for moderate values of N. Optimization problems, such as TSP, naturally lend themselves to parametric studies. There are a large number of instances of TSP, the instances can be systematically varied, and performance can be measured quantitatively by response time and accuracy.

It is worth noting that optimization problems are ubiquitous in cognition. Minimizing a cost function has been a standard way to describe: perception of objects (e.g., Knill & Richards, 1996; Pizlo, 2001), figure-ground organization (Koffka, 1935; Pizlo et al., 1997), decision making and games (e.g., Simon, 1996; von Neuman & Morgerstern, 1944), motor control (e.g., Harris, 1998), categorization (e.g., Nosofsky, 1986), formulating scientific theories (e.g., Li & Vitanyi, 1997; Pitt et al., 2002), as well as human communication (e.g., Quine, 1960). The fact that the human mind optimizes is not surprising considering that the mind is a result of a long evolutionary process, in which the *best* adaptation was achieved when the *best* solutions to everyday life problems were provided. If optimization is the *sine qua non* of cognition, then cognition, including thinking and problem solving, should be studied in optimization tasks.

Finding the shortest TSP tour is difficult because TSP is NP-hard. This means that in the worst case, finding the shortest tour may lead to an exhaustive search through all tours. Because of computational intractability of TSP, there has been growing interest in designing algorithms that can find tours, which are close to the shortest tour, while the time it takes to find the tours does not grow too fast with the problem size N. Review of such algorithms can be found in Lawler et al. (1985) and Gutin & Punnen (2002). Can humans solve TSP well? The answer is in the affirmative. More specifically, humans can produce optimal or near-optimal solutions in a linear time, as long as the problem is presented on a Euclidean plane. A review of recent results on human performance with Euclidean TSP (E-TSP) can be found in the first issue of the *Journal of Problem Solving* (http://docs.lib.purdue.edu/jps/).

The next section presents a brief overview of our model of the mental mechanisms involved in solving E-TSP. The full description can be found in Pizlo et al. (2006). The following two sections present psychophysical experiments and corresponding computational models on human performance in shortest path problem (SPP) and in E-TSP in the presence of obstacles (E-TSP-O). The report is concluded with a summary and suggestions for future research. Our related work on the 15-puzzle, another NP hard problem, has been published by Pizlo & Li (2005). We studied how subjects solve versions of this puzzle having different sizes. We modeled the mental mechanisms by using a graph pyramid model. My work on pyramids had some impact on my theory of shape perception. The theory will be published in a book (Pizlo, 2007). The full list of publications that resulted from this project can be found at the end of this document.

A Pyramid Model for Euclidean-TSP (E-TSP).

Pyramid algorithms have been used extensively to model human visual perception (Jolion & Rosenfeld, 1994; Pizlo et al., 1995, 1997). These algorithms were a natural choice for modeling TSP mental mechanisms because TSP is presented to the subjects as a visual task. Our pyramid model developed for E-TSP works by first generating a multiresolution (pyramid) representation of the problem, and then constructing a tour using this representation and a sequence of top-down refinements. The first version of this model was presented in Graham et al. (2000), and the second version in Pizlo et al. (2006). Both versions received support in experiments, in which subjects solved E-TSP with 6-50 cities. The more recent version of the model is briefly described below.

Construction of the pyramid representation.

Pyramid representation involves a set of "images" that are characterized by more precise, local information on the lower layers of the pyramid and coarser, global information on the top layers of the pyramid. In the pyramid representation the base of the pyramid is the set of points in the problem. This information is exact, but also very local: there is no information about the relationship among the points in the problem. Only the coordinates of each city are stored. In the layer above the base, some points are grouped together to form clusters, which are then abstracted away as a single point, representing the cluster's center of gravity. The exact information about the positions of points is not present at this layer, but new information about the relationship among points has been gained. Near the apex of the pyramid, the problem may be reduced to a handful of clusters, each cluster representing dozens or even hundreds of points. Information about the general shape of the problem replaces information about the exact positions of points.

The first version of the E-TSP algorithm used a bottom-up clustering involving Gaussian blurring (Graham et al., 2000). In that algorithm, the number of clusters on a given layer was not directly controlled and it depended on the distribution of points. In the more recent version, the clustering is top-down and the number of clusters is controlled. Specifically, the n^{th} layer from the apex has 2^{2n} clusters. The child-parent relations between these clusters (regions) on the different layers of the pyramid are fixed Figure 1 illustrates partitioning of cities on several layers of the pyramid representation.

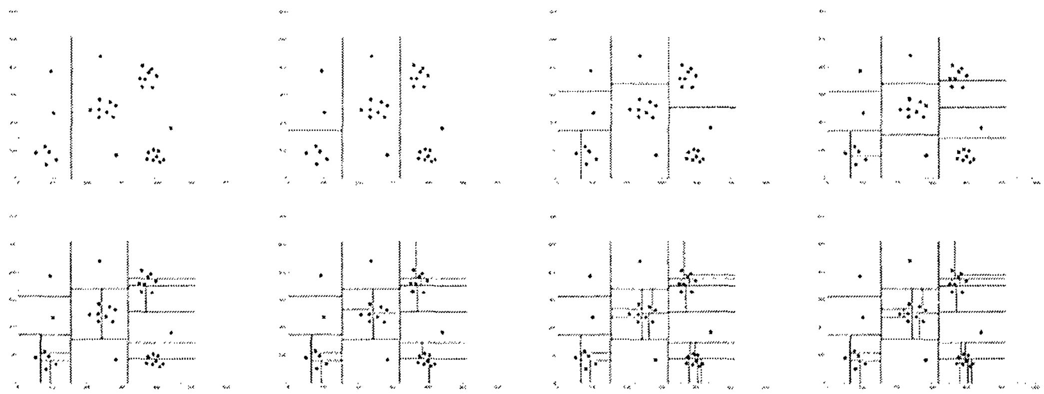

Figure 1. Multiresolution (pyramid) representation of a TSP problem. The red lines represent the boundaries of the clusters.

Construction of the E-TSP tour.

Once the model has generated a pyramid representation of an E-TSP problem, it then takes advantage of the representation to find a good (optimal or near-optimal) tour. Basically, the overall 'shape' of a good TSP tour of the individual points should be an optimal tour of the clusters on the top layers of the pyramid representation. So, the model first finds the best tour of the clusters on the top-most layer of the pyramid, a trivial task as the top-most layer of the pyramid will have a small number of clusters, and then 'refines' this tour. In doing so, the model relies on the fact that, in a pyramid representation, each cluster on any given layer has 'child' clusters on the layer immediately below it in the pyramid. A cluster in the tour is removed, and its children are inserted into the tour near its location. The exact position is determined by local search, specifically local cheapest insertion: for each child cluster, the model considers a small, constant number of positions near the position of the parent node and finds a position that minimizes the length of the tour. The model then inserts the child cluster into the tour at that position. This process of top-down refinement continues until the tour consists of points on the bottom layer of the pyramid.

The tour refinement process incorporates "foveating" and "eye movements". In the first version of the model (Graham et al., 2000), the top-down refinement was carried out one layer at a time. Clusters in the tour which were on a given layer were replaced by their child clusters on the layer below. Once all of the clusters on a given layer had been refined, the refinement moved on to the next layer. But this is not how humans solve the problem. Due to the fact that the distribution of receptors on the retina is not uniform, human subjects move their eye around the problem while solving it. The new version of our model simulates this process. First, the pyramid representation is changed so that only a small region in the problem around model's fovea involves the representation in which individual cities are "visible". For other regions, the highest resolution available is a function of the distance from the fovea. The top-down tour refinement focuses on a specific area of the tour. Refinement moves around the E-TSP instance, so that the model produces a complete, finished part of the tour in one region before moving on to the next (see http://psych.purdue.edu/tsp/workshop/downloads.html for an animation). Figure 2 shows a snapshot of this process. The green part of the tour is represented on the bottom layer of the pyramid and it goes through individual cities. Other parts pf the tour connect centers of gravity of city clusters. This model was shown to provide good fits to the results of several subjects, as measured by the proportion of optimal solutions and the average solution error. The fits are shown in Figure 3. Note that individual variability is quite small. The computational complexity of the model is low (O(NlogN)). Recall that the subjects solve the E-TSP problems in time that is proportional (on average) to the number of cities.

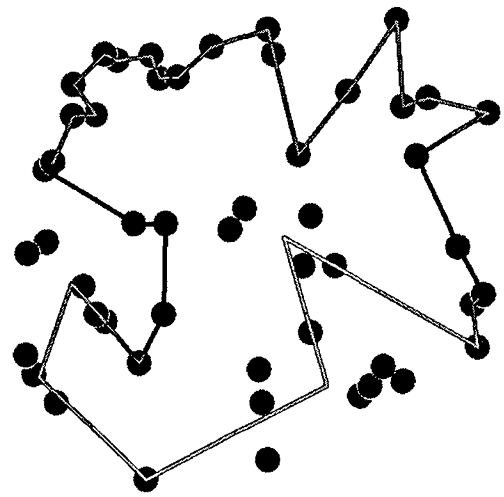

Figure 2. A snapshot from the solution process by the bisection foveating pyramid.

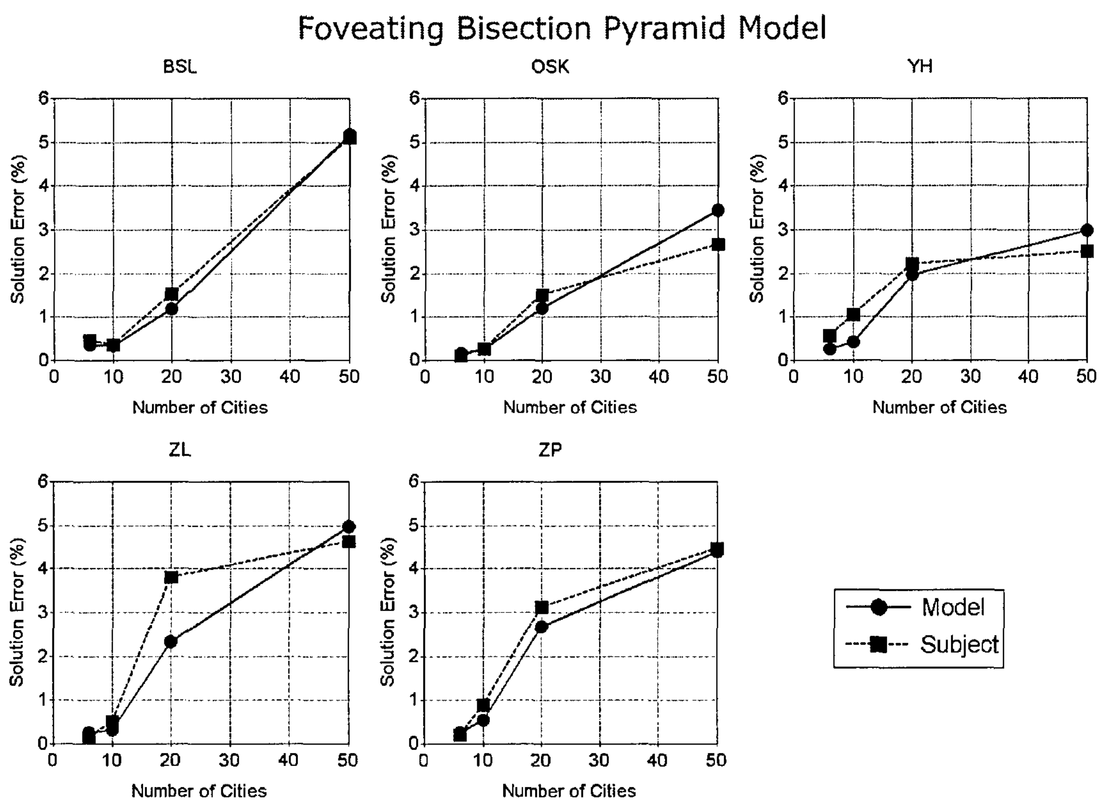

Figure 3a. Model fits to the performance of individual subjects. Average error.

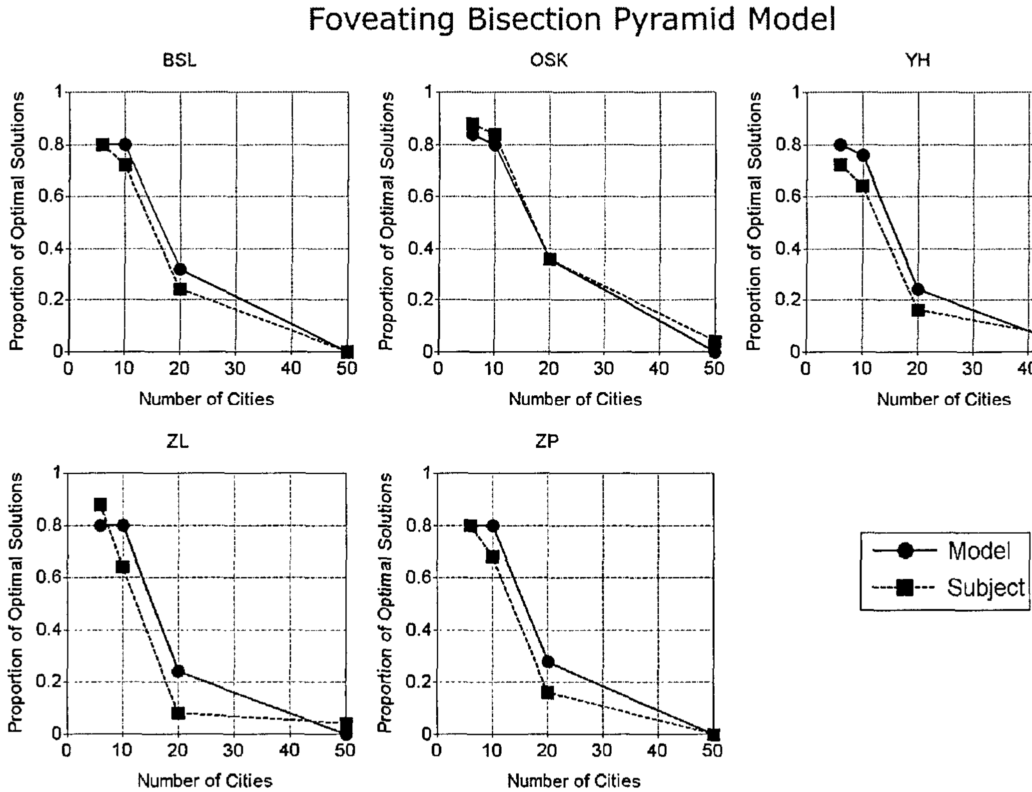

Figure 3b. Model fits to the performance of individual subjects. Proportion of optimal solutions.

The pyramid model was also tested on large problems: 200-1000 cities. The model's average solution is about 10% longer than the shortest tour (see http://psych.purdue.edu/tsp/workshop/ for details). The average time to solve the 1000 city TSP problem is 40 seconds. Clearly, the pyramid model is quite effective and can be used in a number of applications.

Next, I will describe a study that generalized the results and the model to the case of E-TSP in the presence of obstacles (E-TSP-O). Figure 4 illustrates an E-TSP-O problem. The task is to produce a tour in such a way that the tour goes "around" the obstacles. When obstacles are present, the distances between pairs of cities are not Euclidean distances, anymore. So, the problem is not Euclidean. Nevertheless, the distances satisfy metric axioms, so the TSP problem is metric. There are two motivations for studying E-TSP-O. First is to use TSP problems that more closely reflect characteristics of real life problems. Second is to provide additional test of pyramid model. One of the two main aspects of the pyramid model is the use of hierarchical clustering. Clearly, by using obstacles, proximity relations are changed and clusters are modified. Will introducing obstacles make the problem more difficult for the subjects? Will it make the problem more difficult for the model? The model will surely have to be modified. What is the nature of the required modifications?

Intuitively, E-TSP-O seems more difficult than E-TSP. In the extreme case, when obstacles form a maze, as shown in Figure 5, a human subject can no longer cluster points visually: points that are arbitrarily close in the image may have arbitrarily long path between them due to obstacles. In such a case, the only way to proceed is to perform search for paths connecting the pairs of points. Are the shortest paths always found? Are obstacles used at the stage of clustering or at the stage of constructing the tour? The first question is addressed in the next section, and the second question is addressed in the following one.

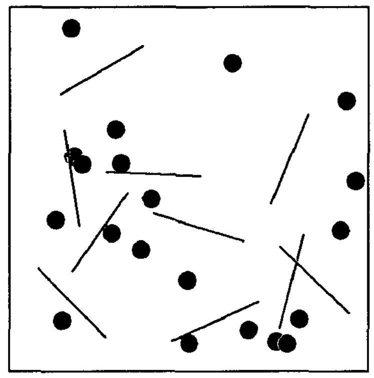

Figure 4. 20 city E-TSP with 10 obstacles.

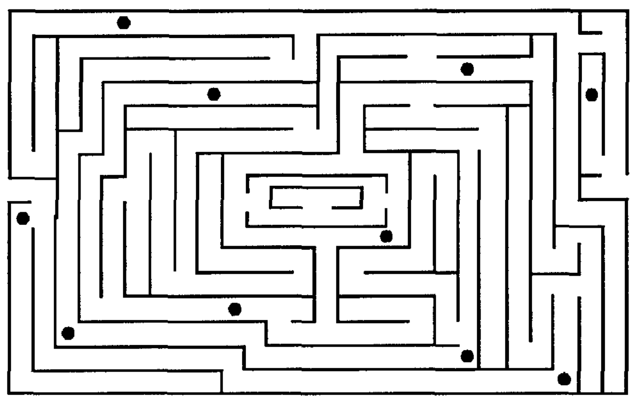

Figure 5. E-TSP in a maze.

Shortest path problem in the presence of obstacles.

Consider the E-TSP-O problem shown in Figure 6. There are eight square-shaped obstacles, each square having two gaps. The gaps are always on opposite sides of a square and the sides (up-down, vs. lefty-right) change from one square to another. This problem resembles the TSP in a maze shown in Figure 5. The optimal solution of this problem is shown as well. It is not obvious to the reader that this solution is indeed optimal. In particular, it is not obvious that the paths involved in this solution are shortest paths for the pairs of cities. They must be; otherwise, the tour would not be the shortest one.

Before we understand and model how humans solve E-TSP in the presence of obstacles, we have to understand how humans find paths between pairs of points in the presence of obstacles. Do they always find the shortest path? If not, what path do they choose?

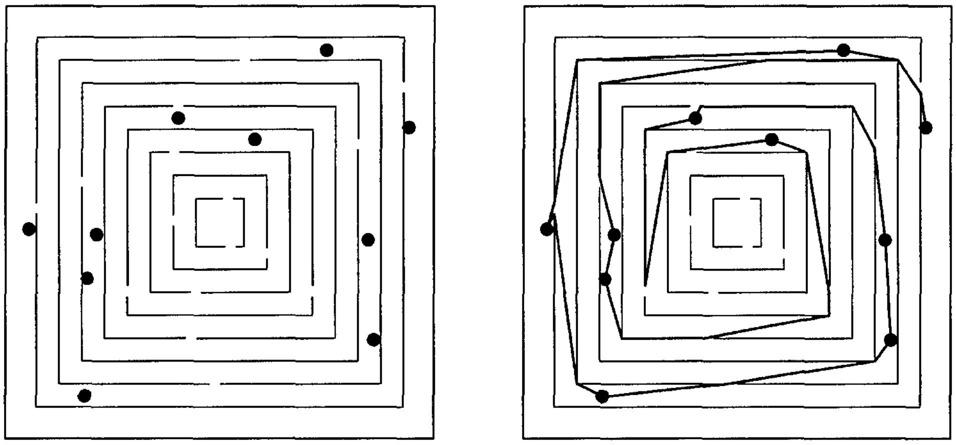

Figure 6. 10 city E-TSP with eight complex obstacles. The optimal tour is shown on the right.

Experiment: Human performance on SPP problem.

Subjects: Two subjects were tested. They ran the same problems in a different and randomly determined order.

Stimuli: The SPP problems were displayed on a computer screen in a window of 512x512 pixels. Purdue TSP Application was used. There were a total of eight sessions representing two types of obstacles (simple vs. complex) and the number of obstacles (1, 2, 4 and 8). There were 25 randomly generated problems per session. Figure 7 illustrates examples of three of these conditions.

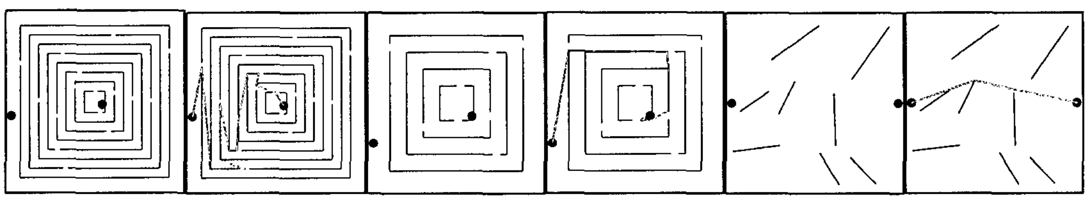

Figure 7. SPP with eight and four complex obstacles, and with eight simple obstacles.

Results.

With complex obstacles the subject had to move and click the mouse more often than with simple obstacles. As a result, the total time to solve the problems was substantially longer with complex obstacles. In order to evaluate the complexity of mental mechanisms not confounded with moving and clicking the mouse, the time per vertex was plotted (see Figure 8). Time per vertex is defined as a total time of solving each problem divided by the number of vertices (including the points representing the start and the goal) in the polygonal line representing the solution tour. Note that the times for both simple and complex obstacles tend to increase with the number of obstacles and the increase rate is somewhat faster with complex obstacles. This fact suggests that the mental mechanisms have average computational complexity higher than linear. This contrasts with E-TSP, where the average time per city does not depend on the number of cities (Pizlo et al., 2006). This comparison has interesting implications. E-TSP is computationally more difficult than SPP. But, mental mechanisms are computationally more complex in the case of SPP than E-TSP. This difference in complexity of mental mechanisms is most likely related to the fact that the human visual system can analyze large parts of E-TSP, but not SPP, in a parallel fashion. When obstacles are present, the shortest path between a pair of cities is not a straight line. Establishing which path is the shortest, involves examining several alternative paths, one after another.

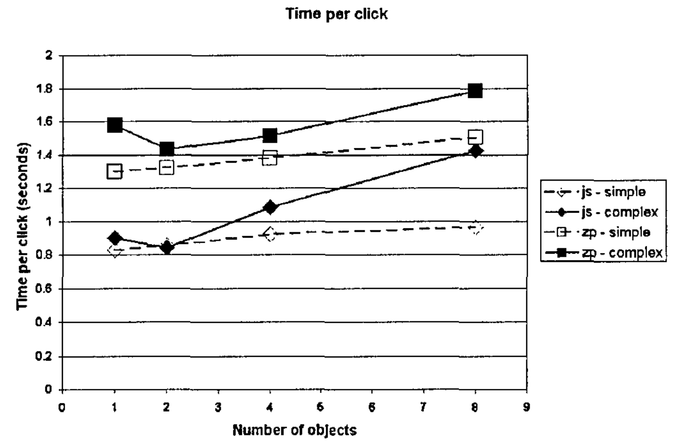

Figure 8. Solution time per vertex in SPP.

The proportion of optimal solutions and the average error are shown in Figure 9. Average error is computed by subtracting the length of the shortest path from the length of the path produced by the subject and normalizing the result to the former. The results for simple obstacles indicate that the subjects are almost always able to produce the shortest paths; the small departures from optimality are likely to result from the visual noise. If the visual system could measure distances precisely, all paths produced by the subjects would have been shortest. This was not the case with complex obstacles. Here, the proportion of optimal solutions dropped, and the average error increased substantially as the number of obstacles increased. Clearly, the subjects were not always able to produce shortest paths in this case. They performed search when they solved the SPP problem, as indicated by the analysis of solution time, but the search was limited and did not guarantee finding the shortest path when complex obstacles were used. This suggests that with complex obstacles, a greedy algorithm was used by subjects, most likely due to the limitations of the short term memory. Specifically, the subjects cannot store all partial paths, as required by the optimal algorithm. The algorithm that guarantees finding the shortest path, as well as a greedy algorithm that does not, are presented in the next section.

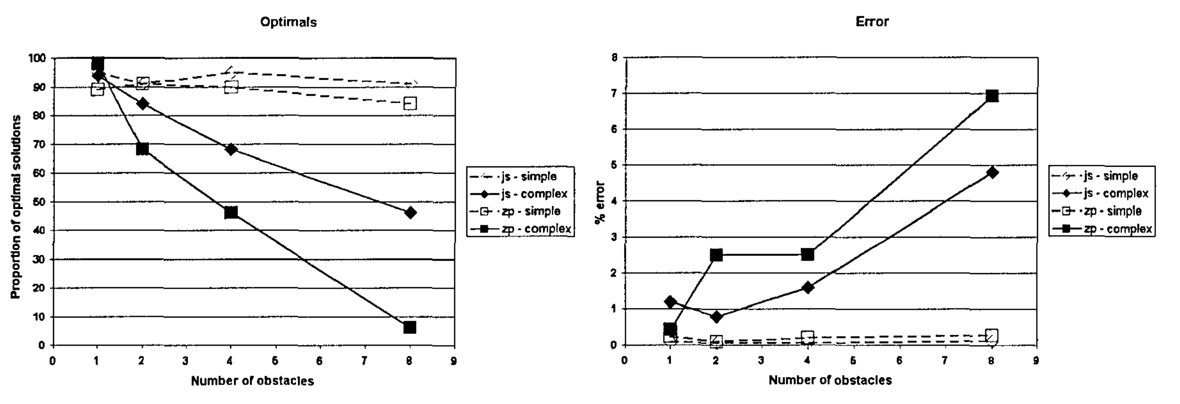

Figure 9. Proportion of optimal solutions (left) and average solution error in SPP (right).

Simulation models for SPP.

First, an algorithm for finding the shortest path is described. We applied Dijkstra's algorithm to the "visibility graph". The visibility graph determines which points in a 2D image are "visible" from any given point. If the point representing the goal is visible from the start point, then the straight line segment connecting these two points is the shortest path. Otherwise, all endpoints of obstacles that are visible from the start point are found and the endpoint that is closest to the start point is selected. Next, all endpoints of obstacles that are visible from the selected point are found, and the shortest path to each of these points from the start point is stored. Again, the endpoint that is closest to the start point is selected. The process is repeated recursively, until the goal is reached. The shortest path from the paths that were stored is guaranteed to be the shortest path from the start to the goal. Recall that the subjects almost always found the shortest path when simple obstacles were used (Figure 9). Subjects' performance can be modeled by this algorithm when the information about the points and obstacles is modified by adding visual noise.

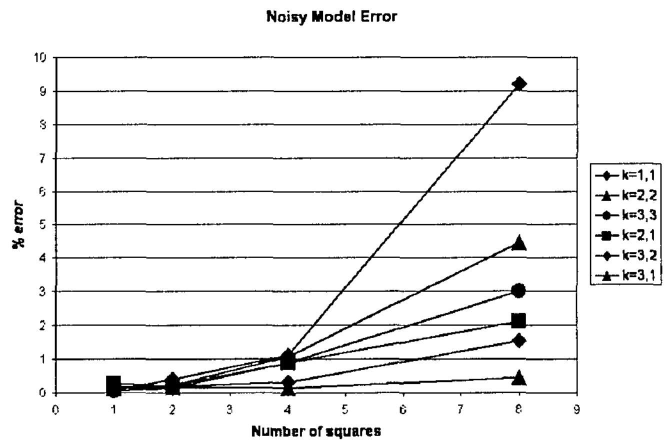

Figure 10. Average error of a greedy SPP model applied to complex obstacles for several levels of local search.

A greedy algorithm that was a modification of the algorithm described above was used to model the subjects' performance with complex obstacles. The visual noise was incorporated in the algorithm by adding a Gaussian noise to every estimate of distance. The mean value of the noise was zero and the standard deviation was 3% of the estimated distance (3% is a Weber fraction in line length discrimination task – Watt, 1987). The algorithm described above was applied to the points representing the start and the goal and to the first k_1 obstacles, counting from the start point. (k_1 represents the limitations of the short term memory.) For example, when $k_1=2$, there are at most four different paths that have to be stored in short term memory). Once SPP for the first k_1 obstacles was determined, the algorithm made k_2 steps to go around the first k_2 obstacles, and the process was repeated by replacing the start point by the point reached after k_2 steps. Obviously, $k_2 \leq k_1$. Figure 10 shows the model's average solution error for several values of k_1, k_2. Figure 11 compares the model's performance for $k_1=k_2=2$ to that of the subjects. Although the fit is not perfect, the graphs show that the effect of the number of complex obstacles on the solution error and proportion of optimal solutions is similar in the case of the model and the two subjects. If the model were fitted to individual problems, the fit would have been substantially better.

Next, the subject's performance in E-TSP-O with three types of obstacles was measured and modeled. The obstacles varied in size and shape.

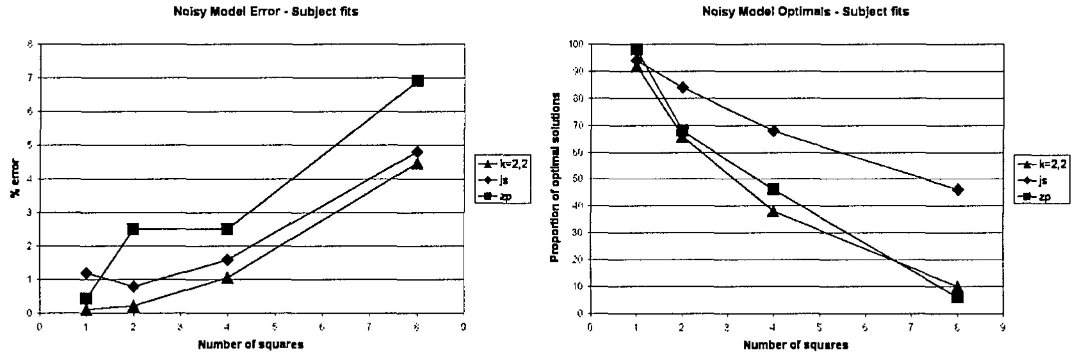

Figure 11. The comparison of the model and the subjects' average errors (left) and proportion of optimal solutions (right) in SPP with complex obstacles.

Euclidean TSP with obstacles (E-TSP-O).

Experiment: Human performance on E-TSP-O.

Subjects. Two subjects were tested. They ran the same problems in a different and randomly determined order.

Stimuli. The TSP problems were displayed on a computer screen in a window of 512x512 pixels. Eight sets of 25 problems were used. Each problem consisted of 20 randomly placed points and 10 randomly placed obstacles. The obstacle length and the obstacle shape varied across sets. Four obstacle lengths were used: 100, 144, 208, and 300 pixels. Three different shapes were used: straight line segments, L shaped and C shaped obstacles (see Figure 12 for examples). If there was an isolated city due to the placement of obstacles, a randomly placed gap was produced in the relevant obstacle to produce a connection to this city.

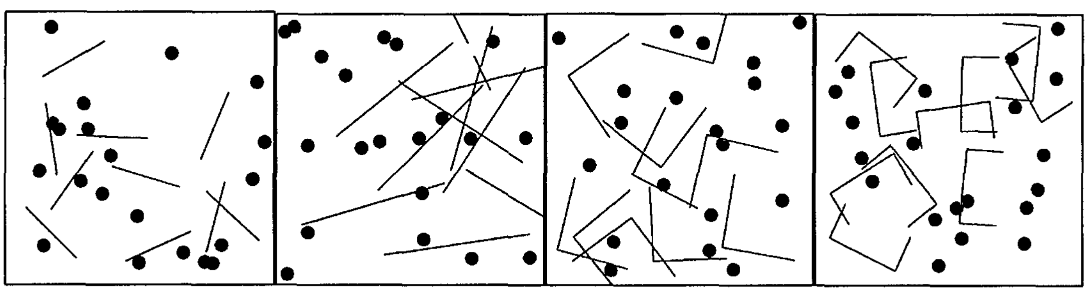

Figure 12. 20 city E-TSP-O with I, L and C obstacles.

Results.

The average solution time per vertex is shown in Figure 13 and the average solution error is shown in Figure 14. The number of vertices in the solution tour is equal to the number of cities plus the number of obstacles whose endpoints had to be included in the solution. The rationale for using time per vertex is the same as that for using time per city in Euclidean TSP. Each is fairly insensitive to the time it takes to move and click the mouse. Longer obstacles are likely to lead to more clicks because more obstacles interfere with the solution tour. The error is computed by subtracting the length of the shortest tour from the length of the subject's tour and normalizing the result to the former.

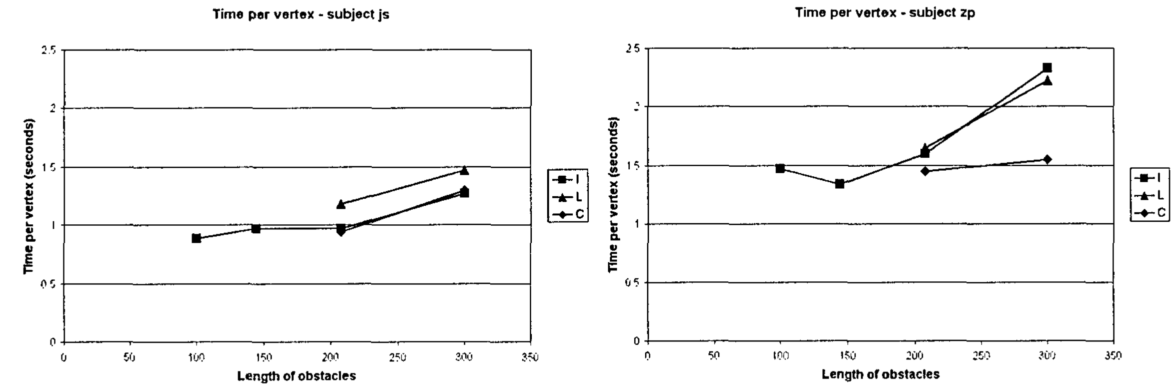

Figure 13. Time per vertex in E-TSP-O.

Solution time was rather systematically affected by the length of the obstacles. It increased from 0.8 to 1.5 sec per vertex in the case of JS and from 1.4 to 2.4 sec per vertex in the case of ZP. For comparison, time per city in the case of 20 city Euclidean TSP was 0.94 sec for JS and 1.21 sec for ZP. The solution error, however, was not systematically affected by the obstacle length. Furthermore, the errors with obstacles were not very different from errors without obstacles. JS's solution error with 20 city Euclidean TSP was 1.2% and ZP's error was 3.1%.

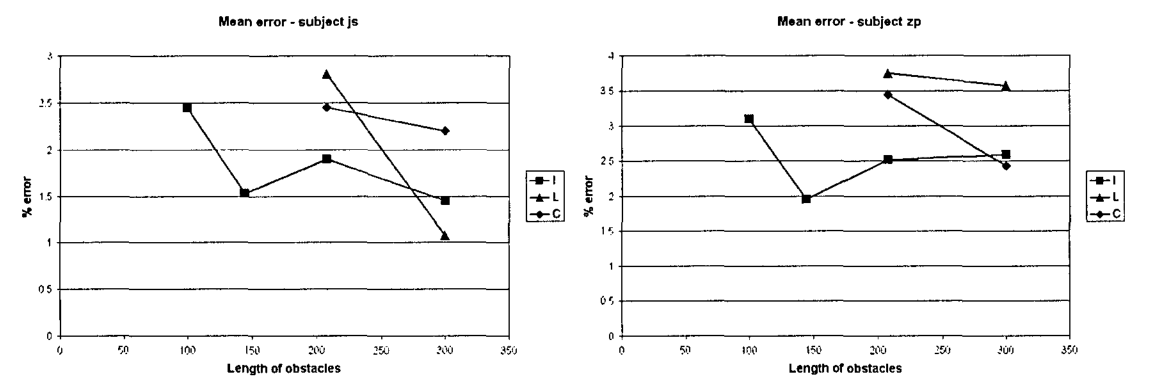

Figure 14. Average error in E-TSP-O.

Discussion.

The effect of the obstacle length on the time per vertex suggests that the complexity of the mental mechanisms increases with an obstacle length. This seems intuitively obvious: longer obstacles force the subject to perform more search. However, once the subject performs search, the tours were not necessarily longer, as measured by the solution error. How should the pyramid model for E-TSP be modified to account for these results on E-TSP-O?

Simulation Models.

A pyramid model for E-TSP developed by Pizlo et al. (2006) was elaborated into a pyramid model for E-TSP-O. Two versions of the E-TSP-O model are presented here. The first version, called Model 1, differed from the E-TSP model only in the way it performed the cheapest insertion during the top-down tour refinement. The hierarchical clustering was performed the same way as in E-TSP of Pizlo et al. (2006). Namely, obstacles were ignored during clustering. They were used only at the second stage when the tour was produced. While performing cheapest-insertion, Model 1 determined and used the shortest path between a given pair of cities. The shortest path was determined by applying Dijkstra algorithm to the visibility graph. The second version of E-TSP-O, called Model 2, is identical to Model 1, except that the obstacles are also used at the stage of clustering. Specifically, after the clusters are formed without obstacles, the shortest paths among centers of gravity of clusters are computed. If a given child node was closer to another node's parent, than to its own parent, the link in the pyramid representation was changed to represent this proximity relation.

Fitting the model to the subject's results was done for each problem individually. Specifically, the model tried all points as starting points as well as both directions of the tour (clockwise and counterclockwise). For each starting point and starting directions the amount of local search was varied by changing the value of parameter k (see Pizlo et al., 2006). This parameter specifies how many nodes were tried in the cheapest insertion method. The tour, whose error was closest to the error of the subject on a given problem, was taken as the best fitting tour.

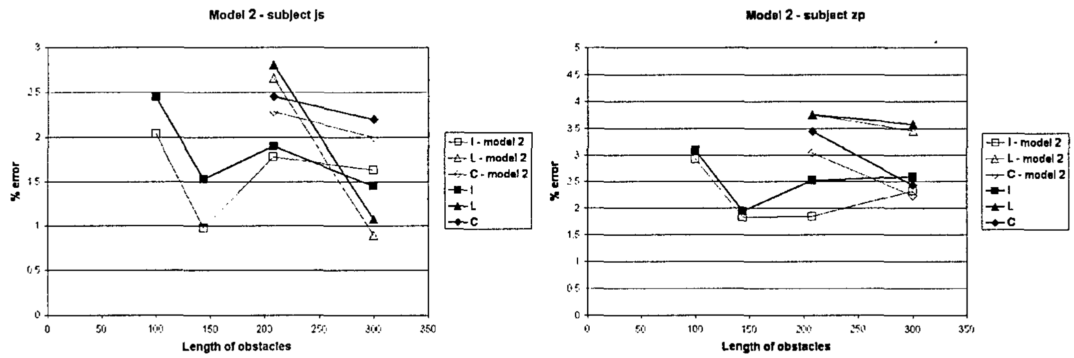

Figure 15. Performance of Model 2 in E-TSP-O.

Figure 15 shows the average solution errors of the best fitting tours for Model 2. The fits by Model 1 were slightly worse. It seems that each model is a possible model of the underlying mental mechanisms. Model 1 can represent trials, in which the subject starts solving the problem without examining the distribution of cities and obstacles in any greater detail. The information about obstacles is taken into account during the solution process. Model 2 can represent trials, in which the subject begins with examining the problem first and determining the actual distances among clusters. Only after the problem is examined, the subject starts producing the tour. Reports of the two subjects, as well as the actual data suggest that both approaches were used. It is unclear at this point how the subject decides to choose which approach (model) is used for a given problem.

Summary and Conclusions

The study on E-TSP-O and SPP showed that humans can find near-optimal solutions to TSP problems not only with Euclidean distances, but also with non-Euclidean ones, when the obstacles are placed on a Euclidean plane. As such, this study generalizes prior results, to TSP problems that are closer to real life applications. In both types of problems, humans can solve the problems quite well without performing exhaustive search. However, the complexity of the mental mechanisms is higher in the case of E-TSP-O due to greater amount search that is needed to establish clusters and/or solve SPP problem. In order to account for subjects' results, an SPP model was formulated and the E-TSP model was elaborated to an E-TSP-O model. With spatially simpler obstacles, the subjects appear to solve the SPP problem optimally. But with more complex obstacles, SPP problem is not solved optimally. In such cases, the subjects use a greedy algorithm, which is likely to lead to larger errors in E-TSP-O.

General Summary

In this project, the subjects were tested in combinatorial problems and demonstrated very good performance. The search spaces were very large making it impossible to represent the whole problem at any one time during solving it. This fact implies that before the subject can start solving a problem, he or she has to construct an effective mental representation. The mental representation was modeled by a multiresolution pyramid. The pyramid algorithm was used to represent a problem and then to provide a solution in a global-to local sequence of approximations.

References

Graham S.M, Joshi A. & Pizlo Z. (2000) The traveling salesman problem: a hierarchical model. *Memory & Cognition, 28*, 1191-1204.

Gutin G. & Punnen A.P. (2002) *The traveling salesman problem and its variations*. Boston: Kluwer.

Harris, C. M. (1998). On the optimal control of behavior: a stochastic perspective. *Journal of Neuroscience Methods 83*, 73-88.

Jolion J.M. & Rosenfeld A, (1994) *A pyramid framework for early vision.* Dordrecht: Kluwer.

Knill, D.C. & Richards, W. (1996) *Perception as Bayesian inference.* Cambridge: Cambridge University Press.

Koffka, K. (1935) *Principles of Gestalt Psychology.* New York: Harcourt, Brace.

Lawler E.L., Lenstra J.K., Rinnooy Kan A.H.G. & Shmoys D.B. (1985) *The traveling salesman problem.* New York: Wiley.

Li, M. & Vitanyi, P. (1997) *An introduction to Kolmogorov complexity and its applications.* New York: Springer.

Von Neumann, J. & Morgernstern, O. (1944) *The theory of games and economic behavior.* Princeton: Princeton University Press.

Nosofsky, R.M. (1986) Attention, similarity and the identification-categorization relationship. *Journal of Experimental Psychology: General, 115*, 39-57.

Pitt, M.A, Myung, J. & Zhang, S. (2002) Toward a method of selecting among computational models of cognition. *Psychological Review, 109*, 472-491.

Pizlo, Z. (2001) Perception viewed as an inverse problem. *Vision Research, 41*, 3145-3161.

Pizlo Z., Rosenfeld A. & Epelboim J. (1995) An exponential pyramid model of the time-course of size processing. *Vision Research, 35*, 1089-1107.

Pizlo, Z., Salach-Golyska, M. & Rosenfeld, A. (1997) Curve detection in a noisy image. *Vision Research, 37*, 1217-1241.

Pizlo, Z., Stefanov, E., Saalwaechter, J., Li, Z., Haxhimusa, Y. & Kropatsch, W.G. (2006) Traveling salesman problem: a foveating algorithm. *Journal of Problem Solving* (in press).

Quine, W.V. (1960) *Word and object.* Cambridge, MA: MIT Press.
Simon, H.A. (1996) *The sciences of the artificial.* Cambridge, MA: MIT Press.

Watt, R.J. (1987) Scanning from coarse to fine spatial scales in the human visual system after the onset of a stimulus. *Journal of the Optical Society of America, A4*, 2006-2021.

Personnel Supported.

Prof. Zygmunt Pizlo (PI)
Zheng Li – undergraduate student, Purdue University.
John Saalweachter – undergraduate student, Purdue University.
Emil Stefanov – undergraduate student, Purdue University.
Prof. Walter Kropatsch – Vienna University of Technology (visit to Purdue University).
Yll Haxhimusa – graduate student from Vienna University of Technology (visit to Purdue University).

List of publications that resulted from this project.

Pizlo, Z. (2007) Shape: its unique place in visual perception. Cambridge, MA: MIT Press (accepted).

Pizlo, Z. & Li, Z. (2003) Pyramid algorithms as models of human cognition. *Proceedings of IS&T/SPIE Conference on Computational Imaging,* vol. 5016, pp. 1-12.

Pizlo, Z. & Li, Z. (2004) Graph pyramid algorithms as models of human problem solving. *Proceedings of IS&T/SPIE Conference on Computational Imaging,* vol.5299, pp.205-15.

Pizlo, Z. & Li, Z. (2005) Solving combinatorial problems: 15-puzzle. *Memory & Cognition* (in press).

Pizlo, Z., Stefanov, E., Saalweachter, J., Li, Z., Haxhimusa, Y. & Kropatsch, W.G. (2006) Traveling Salesman Problem: a Foveating Pyramid Model. *Journal of Problem Solving* (in press).

Kropatsch, W.G., Haxhimusa, Y., Pizlo, Z. & Langs, G. (2005) Vision pyramids that do not grow too high. *Pattern Recognition Letters,***26** , 319-337.

Kropatsch, W.G., Haxhimusa, Y. & Pizlo, Z. (2004) Integral trees: subtree depth and diameter. *Lecture Notes in Computer Science,***33 22**, 77-87. Springer.

Saalweachter, J. & Pizlo, Z. (2007) Non-Euclidean Traveling Salesman Problem. Proceedings of a workshop on Decision Modeling and Behavior in Uncertain and Complex Environments. Springer (to appear).

Software – Purdue TSP Application.

Psychophysical experiments were performed by using the Purdue TSP Application (TSPApp). The demo version of TSPApp can be found at: http://psych.purdue.edu/tsp/workshop/downloads.html. The full version, which is being constantly updated and revised, is available from Z. Pizlo (pizlo@psych.purdue.edu). TSPApp is a program written in C++, which allows generating TSP problems, testing human subjects, determining the optimal tour and running simulations with our TSP model. TSPApp also includes functions for testing the minimum spanning tree (MST) and shortest path (SPP) problems. It allows generating Euclidean TSP problems, as well as TSP problems with obstacles. The obstacles vary with respect to size and shape.

Final Report

Workshop on Human Problem Solving:
Difficult Optimization Tasks
Zygmunt Pizlo and Edward Chronicle

Project Activities and Findings

The major activity involved organizing a workshop, running it and disseminating the presentations. An important focus was to facilitate interactions amongst international researchers working in the same area: human performance with optimization problems. No previous meeting has permitted such interactions. A number of collaborative relationships have formed as a result of this workshop.

Outreach Activities

The information about the workshop, including the talks and selected software are available on the workshop web site:

http://psych.purdue.edu/tsp/workshop/

After the workshop, a mailing list and a web site for those interested in problem solving were setup. The url of the web site is as follows:

http://spiderman.psych.purdue.edu/problem_solving/people/

There are currently 30 names on the web site. The mailing list itself, contains 90 persons. Not everyone from those on the mailing list put their contact information on the web site.

A new electronic journal, called the Journal of Problem Solving (JPS) was started:

http://docs.lib.purdue.edu/jps/

This is the first journal dedicated to human problem solving. The first issue will appear this Fall.

Publications and Products

Chronicle, E.P. & Pizlo, Z.P. Human performance with optimization problems: when, how and why (in preparation).

Contributions within Discipline

This workshop helped to initiate a new interest in human problem solving. Specifically, it helped identify researchers who are interested in studying how humans solve difficult combinatorial problems. Combinatorial problems are of special interest because they are 'computationally intractable', and yet, humans are able to provide very good solutions quickly. Understanding the underlying mental mechanisms is of fundamental importance for psychology of problem solving and decision making in particular, and human cognition in general.

Contributions to Other Disciplines

The workshop attracted not only psychologists but also researchers in computer science, artificial intelligence and operations research. Clearly, the fact that human mind involves some powerful combinatorial optimization mechanisms is of interest to those computer scientists and engineers who are trying to formulate better and better algorithms for solving difficult problems. The workshop, the mailing list and the new journal are likely to establish a multidisciplinary collaboration between psychology and AI, a collaboration that hardly existed during the last 50 years.

Contributions to Human Resource Development

As already indicated, the workshop led to founding a new journal and setting up a mailing list. It is expected that a new society will be formed with regular conferences. In the anticipation of the regular conferences, one of the PIs (ZP) organized a special symposium at the 2006 Annual Meeting of the Society for Mathematical Psychology:

http://www.cogs.indiana.edu/socmathpsych/meetings.html#socmeet

Contributions Beyond Science and Engineering

Understanding mental mechanisms that underlie problem solving is likely to improve the methods that are used to teach how to solve problems. This will be very important on all levels of education: from kindergarten to college and graduate school. Furthermore, it will allow improving the working environment. In particular, in deciding how a given workload should be divided between a human and a computer. This will lead to efficient human-computer interaction systems.